Lec 7

$$Y_t = Y_{t-1} + \epsilon_t \qquad \epsilon_t \sim N(0,1)$$

$$Y_0 = 0$$

$$E(Y_t) = E\left(\sum \epsilon_i\right) = \sum \left(E\epsilon_i\right) = \sum 0 = 0$$

$$Y_0 = 0$$

$$Y_1 = \epsilon_1$$

$$Y_2 = \epsilon_2 + \epsilon_1$$

$$Y_3 = \epsilon_3 + \epsilon_2 + \epsilon_1$$

$$\vdots$$

$$Y_t = \sum_{i=1}^{t} \epsilon_i$$

$$Cov\left(Y_t, Y_{t+k}\right) = \frac{E\left(\left(Y_t - 0\right)\left(Y_{t+k} - 0\right)\right)}{\sqrt{Var(Y_t)\ Var(Y_{t+k})}}$$

$$= E\left(Y_t \cdot Y_{t+k}\right)$$

$$= E\left(\left(\sum_{i=1}^{t} \epsilon_i\right)\left(\sum_{j=1}^{t+k} \epsilon_j\right)\right)$$

$$= t$$

if $i = j$

$$E\left(\epsilon_i\ \epsilon_j\right) = E\left(\epsilon_i^2\right)$$

$$= Var\left(\epsilon_i\right) + E\left(\epsilon_i\right)^2 = 1 + 0 = 1$$

if $i \neq j$

$$E\left(\epsilon_i\ \epsilon_j\right) = E\left(\epsilon_i\right)E\left(\epsilon_j\right)$$

$$= 0 \cdot 0 = 0$$

Pick $y_t = \delta + y_{t-1} + w_t \quad y_0 = 0$

$y_0 = 0$

$y_1 = \delta + w_1$

$y_2 = 2\delta + w_2 + w_1$

$y_3 = 3\delta + w_3 + w_2 + w_1$

$\vdots$

$y_t = t\delta + \sum_{i=1}^{t} w_i$

$E(y_t) = \delta t$

$Cov(y_t, y_{t+k}) = t$

MA $\quad Y_t = v_{t-1} + w_t \qquad v_t \sim N(0,1)$

$$Y_1 = v_0 + w_1$$
$$Y_2 = w_1 + w_2$$
$$Y_3 = w_2 + w_3$$
$$Y_4 = v_3 + v_4$$
$$\vdots$$
$$Y_t = v_{t-1} + v_k$$

$$E(Y_t) = E(v_{t-1}) + E(v_t)$$
$$= 0 + 0 = 0$$

$$Cov(Y_t, Y_{t+k})$$
$$= \frac{E((Y_t - 0)(Y_{t+k} - 0))}{\sqrt{Var(Y_t)\, Var(Y_{t+k})}}$$

$$= \frac{1}{2} E(Y_t\, Y_{t+k})$$

$$= \frac{1}{2} \begin{cases} 2 & k = 0 \\ 1 & k = \pm 1 \\ 0 & |k| \geq 2 \end{cases}$$

$$Y_k \cdot Y_{t+1} = (v_t + v_{t-1})(v_{t+1} + v_k)$$
$$= 1$$